Lesson 19 (3.11) → Not in Exam 2

Today: Related Rates

Office Hours: M, W, F : 2:45PM – 4:15PM (MATH 842)

Announcements: * Exam 2: Wednesday, Oct 15, 8PM-9PM } Instructions on Brightspace

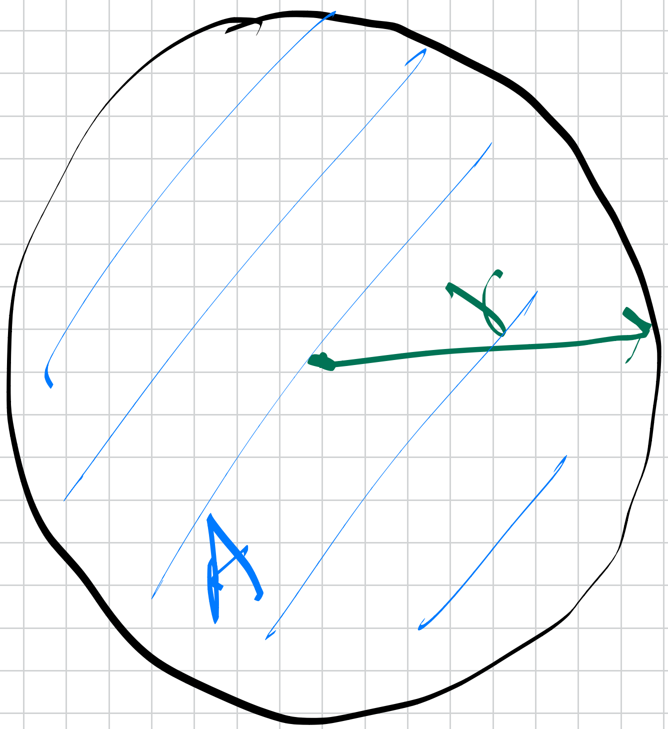
* No Classes on Monday and Tuesday

Related Rates

- Variable are functions of time / Changing with time
- Find rate at which something is changing given something else is changing.

1. Read and Understand question Carefully
2. Draw a picture + label variables
3. Write: What is known / Given
   or identify What is unknown / what to find.
4. Write an equation relating known & unknown.
5. $\frac{d}{dt}$ of the equation, plug in knowns & find unknowns

How fast is the area of a circle changing if the radius of the circle is increasing at the rate of 2cm/sec when the radius is 4 cm.



Given: $\frac{dr}{dt} = 2 \text{ cm/sec}$.

Unknown/find: $\frac{dA}{dt}$ when $r = 4\text{cm}$.

Equation relating A & r: $A = \pi r^2$

take $\frac{d}{dt}$: $\frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = 2\pi r \frac{dr}{dt}$

$$\left.\frac{dA}{dt}\right|_{r=4\text{cm}} = 2\pi \, 4\text{cm} \cdot 2\text{cm/sec} = 16 \text{ cm}^2/\text{sec}$$

increasing $\rightsquigarrow$ rate of change is > 0

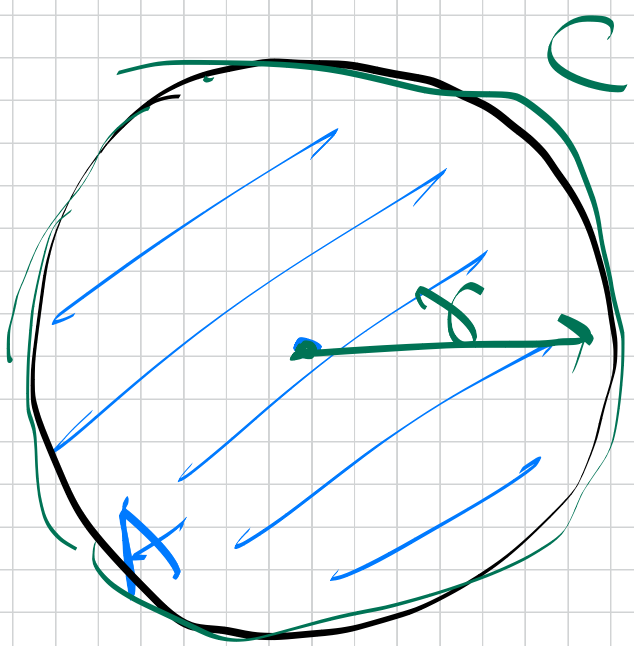
decreasing $\rightsquigarrow$ rate of change is < 0

$\frac{dr}{dt}$ $\rightsquigarrow$ increasing at $\left.2cm/s\right\}$ $\frac{dr}{dt} = 2cm/sec$

$\frac{dA}{dt} = 16\pi \ cm^2/sec > 0$

$\hookrightarrow$ Area is increasing.

How fast is the area of a circle changing if the circumference of the circle is increasing at the rate of 2cm/sec when the radius is 4 cm.



Given! $\frac{dc}{dt} = 2$ cm/sec

$r = 4$cm ⇝ $C = 2\pi r = 8\pi$ cm

directly given

indirectly given

Unknown! $\frac{dA}{dt}$

Equation relating A & C : Make use of

$A = \pi r^2$

$C = 2\pi r \rightarrow r = \frac{c}{2\pi}$

$A = \pi \cdot \frac{c^2}{4\pi^2} = \frac{c^2}{4\pi}$

$$\boxed{A = \frac{c^2}{4\pi}}$$

Equation 1 $\quad A = \dfrac{C^2}{4\pi}$

$\dfrac{d}{dt}$ on Both sides $\quad \dfrac{dA}{dt} = \dfrac{1}{4\pi} \dfrac{d}{dt} C^2 = \dfrac{1}{4\pi} * 2C * \dfrac{dC}{dt}$

$$\dfrac{dA}{dt} = \dfrac{C}{2\pi} \dfrac{dC}{dt}$$

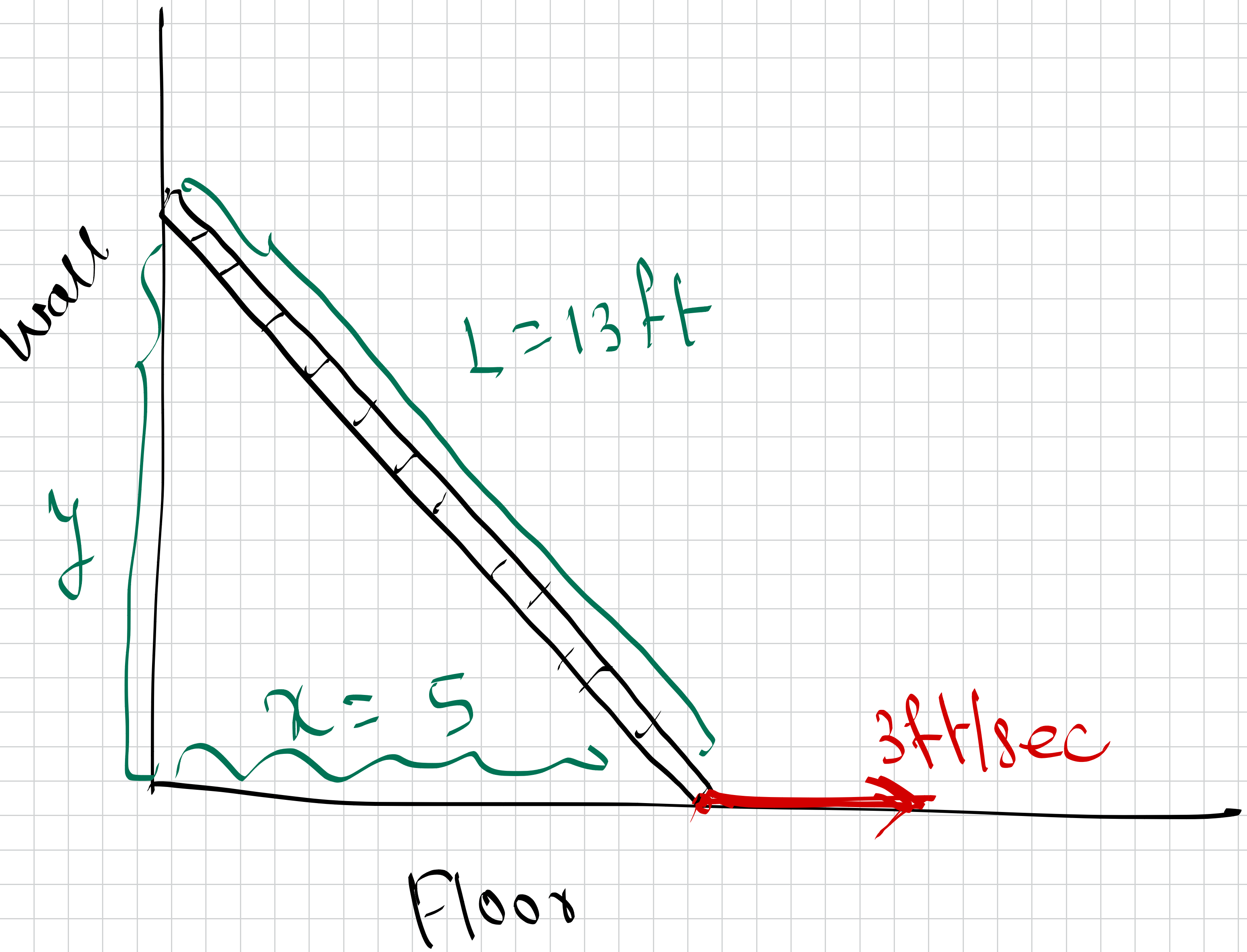
$\dfrac{dC}{dt}$ → 2 cm/sec.

$C = 8\pi$ cm

when $r = 4$ cm

$$\left.\dfrac{dA}{dt}\right|_{r = 4\text{ cm}} = \dfrac{8\pi\text{ cm}}{2\pi} * 2\text{ cm/sec} = \underline{\underline{8\text{ cm}^2/\text{sec}}}$$

A 13 foot ladder leans against a tall wall. If the bottom is pulled away from the wall at 3ft/sec, how fast is the top of the ladder sliding down the wall when the bottom is 5 ft from the wall.



Given: $\frac{dx}{dt} = 3\,ft/sec$ when $x = 5\,ft$

Unknown: y when $x = 5$

$\frac{dy}{dt}$

Equation Relating x & y:

$$x^2 + y^2 = L^2 = 13^2 = 169$$

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Equation 1 $x^2+y^2=169$

find $\frac{dy}{dt}$, given $\frac{dx}{dt}=3ft$, when $x=5ft$

$\frac{d}{dt}$ on Both sides

$$\frac{d}{dt}(x^2+y^2)=\frac{d}{dt}169$$

$$2x\frac{dx}{dt}+2y\boxed{\frac{dy}{dt}}=0$$

5 3 ? 12 ft

What is y when x=5?

$x^2+y^2=169$

$25+y^2=169$

$y^2=144$

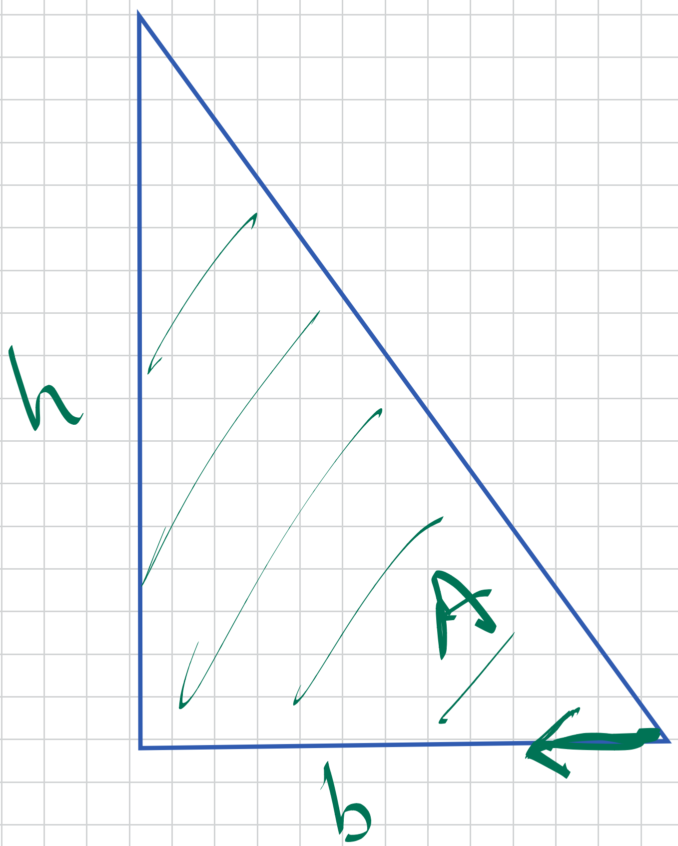
$y=12ft$

$$2*5ft*3ft/sec+2*12ft.\frac{dy}{dt}=0$$

$$\frac{dy}{dt}=-\frac{15}{12}ft/sec.$$

- y is decreasing
- Distance b/w top of ladder & floor is decreasing.

If the base of a right angled triangle is decreasing at 2cm/sec when the base is 5cm and the height is 4cm. How should the height change for the area of the triangle to remain unchanged.



Given: $\frac{db}{dt} = -2\ cm/sec$,

$A =$ constant $\Rightarrow \frac{dA}{dt} = 0$

Find: $\frac{dh}{dt}$ when $b = 5$, $h = 4$,

Equation Relating base, height & Area: $A = \frac{1}{2} bh$

$\frac{d}{dt}$ on Both Sides: $\underbrace{\frac{dA}{dt}}_{0} = \frac{1}{2}\left[\underbrace{\frac{db}{dt}}_{-2}\underbrace{h}_{4} + \underbrace{b}_{5} \cdot \frac{dh}{dt}\right]$

$$0 = \frac{1}{2}\left[-8 + 5\frac{dh}{dt}\right]$$

$$\Rightarrow \frac{dh}{dt} = 8/5\ cm/sec.$$